

Research on Decision-Making Model of Two-Dimensional Product Warranty Service under Time Constraints

Qian Wang^{1,2, a}, Zhonghua Cheng^{1, b, *}, Yongsheng Bai^{1, c}, Qiang Wang^{1, d}, and Xing Song^{1, e}

¹Department of Equipment Command and Administration, Shijiazhuang Campus of Army Engineering University, Shijiazhuang, China

²Ninth comprehensive training base of Army, Zhangjiakou, China

^a540841926@qq.com, ^b18003131595@163.com, ^cxiaobai2004@sohu.com, ^d511091065@qq.com, ^eflying506@qq.com

*Corresponding author

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Abstract: In order to win the favour of customers during the warranty period, it is a common strategy for manufacturers to provide preventive maintenance for some expensive products. How to provide reasonable warranty service to customers more practically is a problem that manufacturers must consider. This paper proposes to consider the time constraints of warranty service. Under the incomplete preventive maintenance strategy, the time of preventive maintenance during the warranty period is limited. Based on the relevant theory of product reliability, a two-dimensional product maintenance cost model under is established to determine how to carry out preventive maintenance. The validity of the model is verified by an example, which provides a basis for manufacturers and customers to make two-dimensional warranty service decisions.

1. Introduction

In the fierce market competition, in addition to reasonable price, excellent performance and excellent quality, after-sale warranty service also plays an increasingly important role in product marketing. During the warranty period, good after-sales service has become an important factor in competition among enterprises. According to the deadline of warranty service, the warranty service can be divided into one-dimensional warranty, two-dimensional warranty and multi-dimensional warranty strategy [1]. At present, two-dimensional warranty strategy has been widely used in many military and civil equipment maintenance services. Many military and civilian general vehicles are guaranteed by using time and driving mileage.

In the current warranty service, the maintenance cost during the warranty period is usually borne by the manufacturer, that is, after the product breaks down, the manufacturer carries out free maintenance. In order to reduce the cost of warranty, the warranty service has gradually changed from a repair-oriented maintenance strategy to a preventive maintenance strategy. Many scholars at home and abroad have proved the superiority of this strategy through research. Huang establishes a cost model under periodic preventive maintenance strategy, and obtains a maintenance plan that maximizes the manufacturer's profit [2]. Wu studied the optimization of periodic preventive maintenance considering aging loss during warranty service period [3]. Huang analyzed the two-dimensional warranty cost under the periodic preventive maintenance strategy [4]. Li establishes two-dimensional single-component and multi-component maintenance service decision-making model, calculates the optimal preventive maintenance interval, and proves the practicability and effectiveness of the model through an example [5]. Xu put forward the non-periodic preventive maintenance model. Through comparison, the model has lower maintenance cost than the periodic preventive maintenance model, and also lower maintenance cost than the minimum maintenance model [6]. Aiming at the lowest cost of manufacturer's quality assurance, Dai establishes an optimization model of flexible preventive maintenance strategy, and reasonably determines the

optimal preventive maintenance times and degree within the scope of quality assurance [7]. Kuang establishes a preventive maintenance model with incomplete period during the warranty period, and the degree of preventive maintenance increases. Finally, an example is given to verify the applicability of the model in a certain range [8]. Husniah divided the two-dimensional warranty period into five stages, and preventive maintenance is carried out only in two stages [9]. Nasrum studied the two-dimensional warranty contract of dump trucks, and carried out periodic preventive maintenance in the two-dimensional warranty. Two warranty strategies are provided and the calculation method of the optimal preventive maintenance cycle is given [10].

In the current research on preventive maintenance strategy, models are built under the most ideal assumptions, without considering the time constraints of preventive maintenance. Such theoretical research does not match the actual situation in engineering applications. Through consulting a large number of literatures related to the study of warranty service, no relevant research with time constraints has been found. This paper is the first time to propose a decision-making model of maintenance service considering preventive maintenance under time constraints. The proposed time constraint mainly refers to the time limit for preventive maintenance, which mainly comes from users, manufacturers and natural environment. Only in a certain period of time can preventive maintenance be carried out. The new model can solve the practical problems existing in the current warranty service to a certain extent. It can be used for reference to reduce the cost of large-scale equipment warranty period. The most important thing is to reduce the price of products and enhance the attractiveness of customers. It is very beneficial to both manufacturers and users. For large equipment, to achieve complete maintenance is not practical, preventive maintenance is usually incomplete maintenance. Therefore, this paper studies the decision-making model of equipment maintenance service under incomplete preventive maintenance strategy.

2. Incomplete Preventive Maintenance Strategy

Incomplete maintenance can be divided into incomplete repairing maintenance and incomplete preventive maintenance. Incomplete preventive maintenance can restore the malfunctioning product. However, due to the limitations of maintenance equipment, maintenance capacity and maintenance personnel, the product performance status will not be restored as new. Preventive maintenance of large equipment is usually incomplete maintenance. In order to describe the change of product failure rate before and after incomplete preventive maintenance, a repair factor δ is introduced. Assuming preventive maintenance at T , the performance of the product is improved and the failure rate is reduced to the same at δT . From the recurrence relation, the failure rate of the product can be obtained as follows:

$$\begin{aligned}\lambda_0(t) &= \lambda(t) \\ \lambda_1(t) &= \lambda(t - \delta T_1) \\ \lambda_2(t) &= \lambda(t - \delta(T_1 + T_2)) \\ &\dots \\ \lambda_i(t) &= \lambda(t - \delta(T_1 + T_2 + \dots + T_i))\end{aligned}$$

In the above formula, $\lambda_i(t)$ is the failure rate of the product after the i th incomplete preventive maintenance. T_i is the interval between the $i-1$ th and i th preventive maintenance.

3. Maintenance cost per unit time model for products in Two-dimensional warranty with time constraints

3.1 Model hypothesis

a) After incomplete preventive maintenance, the failure rate of the product is between the repair as new and the repair as old, and the failure rate of the product does not change after the minimum maintenance.

- b) The product has aging characteristics, and the failure rate increases with the increase of service time and degree.
- c) During the two-dimensional warranty period, under the condition that the preventive maintenance time is limited, the manufacturer carries out preventive maintenance on the products.
- d) Each preventive maintenance has the same impact on product failure rate.
- e) Maintenance time is far smaller than warranty time. When establishing model, maintenance time can be neglected.
- f) Only within each time constraint period, preventive maintenance can be performed.

3.2 Establishment of Two-dimensional Warranty maintenance cost per unit time Model

In the two-dimensional warranty period, the maintenance cost model for preventive maintenance with n time constraints is established. The time limit and usage limit of two-dimensional warranty period are W_0 and U_0 , respectively. The warranty period ends when the product is used longer than W_0 or more than U_0 . Assuming that the usage rate of a single product remains unchanged during the warranty period and different users have different usage rates, for batch products, the usage rate is a random variable. The distribution function of usage is $G(r)$, and probability density function of usage is $g(r)$. The failure rate function is $\lambda(t|r)$ when the product usage rate is r . Expression is: $\lambda(t|r) = \theta_0 + \theta_1 r + \theta_2 t^2 + \theta_3 r t^2$ [11]. According to the climatic conditions in the area where the product is used, and through the agreement between the manufacturer and the user, it is determined that only the n intervals, $[T_{a1}, T_{b1}]$, $[T_{a2}, T_{b2}]$, ..., $[T_{an}, T_{bn}]$, the manufacturer can carry out n time's preventive maintenance for the products. The degree of preventive maintenance is incomplete maintenance, the repair factor is δ , and the cost of each preventive maintenance is C_p . After the product breaks down, only the smallest maintenance is carried out, and the cost is C_f . Under this assumption, during the interval of preventive maintenance, the equipment failure obeys the non-homogeneous Poisson process (NHPP). Assume that n preventive maintenance times are $T_1, T_2, \dots, T_n, \dots$, respectively, $T_1 \in [T_{a1}, T_{b1}]$, $T_2 \in [T_{a2}, T_{b2}]$, ..., $T_n \in [T_{an}, T_{bn}]$, as shown in figure 1.

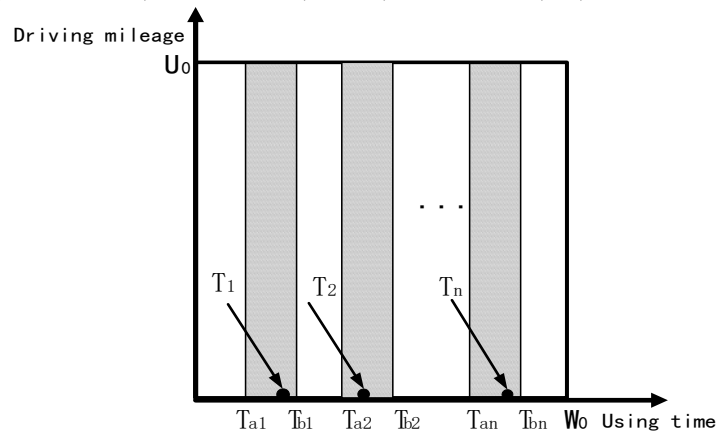


Figure 1. Preventive Maintenance under Time Constraints.

Due to the different utilization rate of equipment, the cost of n preventive maintenance should be divided into (n + 2) cases.

- a) When $r > \frac{U_0}{T_1}$, no preventive maintenance is carried out, and the warranty service is terminated at the end of the driving mileage. The maintenance cost per unit time is as follows:

$$E_1(C) = \frac{C_f \int_0^{U_0} r \lambda(t|r) dt}{\frac{U_0}{r}} \tag{1}$$

b) When $\frac{U_0}{T_2} < r < \frac{U_0}{T_1}$, a preventive maintenance is carried out and the warranty service is terminated at the end of the driving mileage. The maintenance cost per unit time is as follows:

$$E_2(C) = \frac{C_f \int_0^{T_1} \lambda(t|r) dt + C_f \int_{T_1}^{\frac{U_0}{r}} \lambda(t - \delta T_1 | r) dt + C_p}{\frac{U_0}{r}} \quad (2)$$

c) When $\frac{U_0}{T_n} < r < \frac{U_0}{T_{n-1}}$, n-1 times preventive maintenance are carried out and the warranty service is terminated at the end of the driving mileage. The maintenance cost per unit time is as follows:

$$E_n(C) = \frac{C_f \sum_{i=1}^{n-1} \int_{T_{i-1}}^{T_i} \lambda(t - \delta T_{i-1} | r) dt + C_f \int_{T_{n-1}}^{\frac{U_0}{r}} \lambda(t - \delta T_{n-1} | r) dt + (n-1) C_p}{\frac{U_0}{r}} \quad (3)$$

d) When $\frac{U_0}{W_0} < r < \frac{U_0}{T_n}$, n times preventive maintenance are carried out and the warranty service is terminated at the end of the driving mileage. The maintenance cost per unit time is as follows:

$$E_{n+1}(C) = \frac{C_f \int_0^{T_1} \lambda(t|r) dt + C_f \int_{T_1}^{T_2} \lambda(t - \delta T_1 | r) dt + \dots + C_f \int_{T_{n-1}}^{T_n} \lambda(t - \delta T_{n-1} | r) dt + C_f \int_{T_n}^{\frac{U_0}{r}} \lambda(t - \delta T_n | r) dt + n C_p}{\frac{U_0}{r}} \quad (4)$$

$$= \frac{C_f \sum_{i=1}^n \int_{T_{i-1}}^{T_i} \lambda(t - \delta T_{i-1} | r) dt + C_f \int_{T_n}^{\frac{U_0}{r}} \lambda(t - \delta T_n | r) dt + n C_p}{\frac{U_0}{r}}$$

e) When $r < \frac{U_0}{W_0}$, n times preventive maintenance are carried out and the warranty service is terminated at the end of the driving mileage. The maintenance cost per unit time is as follows:

$$E_{n+2}(C) = \frac{C_f \int_0^{T_1} \lambda(t|r) dt + C_f \int_{T_1}^{T_2} \lambda(t - \delta T_1 | r) dt + \dots + C_f \int_{T_{n-1}}^{T_n} \lambda(t - \delta T_{n-1} | r) dt + C_f \int_{T_n}^{W_0} \lambda(t - \delta T_n | r) dt + n C_p}{W_0} \quad (5)$$

$$= \frac{C_f \sum_{i=1}^n \int_{T_{i-1}}^{T_i} \lambda(t - \delta T_{i-1} | r) dt + C_f \int_{T_n}^{W_0} \lambda(t - \delta T_n | r) dt + n C_p}{W_0}$$

To sum up, the maintenance cost per unit time of equipment warranty service during the warranty period is as follows:

$$C(T_1, T_2, \dots, T_n) = \int_{\frac{U_0}{T_1}}^{\infty} E_1(C) g(r) dr + \int_{\frac{U_0}{T_2}}^{\frac{U_0}{T_1}} E_2(C) g(r) dr + \dots$$

$$+ \int_{\frac{U_0}{T_n}}^{\frac{U_0}{T_{n-1}}} E_n(C) g(r) dr + \int_{\frac{U_0}{T_n}}^{\frac{U_0}{W_0}} E_{n+1}(C) g(r) dr + \int_0^{\frac{U_0}{T_n}} E_{n+2}(C) g(r) dr \quad (6)$$

4. Example analysis

It is known that the two-dimensional warranty period of a general equipment is 3years or 160.000 km in the warranty contract. After consultation between the manufacturer and the customer, only in the two periods from December of the first year to February of the second year and December of the second year to February of the third year can the manufacturer and the customers carry out two preventive maintenance of the equipment. Preventive maintenance is incomplete maintenance. Minimum maintenance is used for equipment failure in the rest of the time. According to the statistical analysis of the historical data of the utilization rate of the same type of equipment, the

utilization rate r of the equipment obeys the Weibull distribution. The scale parameter is 1.2 and the shape parameter is 2. Other parameter settings are shown in table 1.

Table 1. Parameter settings.

parameter	parameter values
δ	0.7
C_p/yuan	280
C_f/yuan	500
$\theta_0, \theta_1, \theta_2, \theta_3$	$3 \times 10^{-7}, 2 \times 10^{-7}, 1 \times 10^{-7}, 2 \times 10^{-7}$

In order to simplify the calculation, in the three-year warranty period, the monthly calculation is based on 30 days, a total of 1080 days in three years. The time constraints of two preventive maintenance are [330, 420], and [690, 780].

Using MATLAB tool software and numerical algorithm, T_1 takes value in [330, 420] and the step length takes 5 (days), while T_2 takes value in [690, 780] and the step length was 5 days. The corresponding costs of each group are calculated separately, and the results are shown in figure 2. It can be seen that under different combinations of T_1 and T_2 , the maintenance cost per unit time is different. The calculated cost surface has the lowest point, which indicates that there is the best T_1 and T_2 to minimize the maintenance cost per unit time during the warranty period.

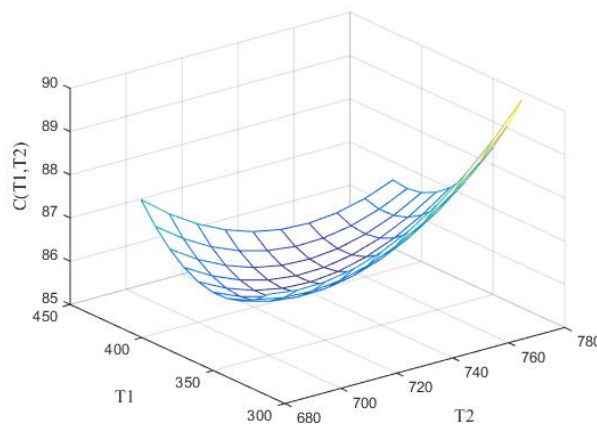


Figure 2. Maintenance cost per unit time during the warranty period.

Through calculation, it is determined that the corresponding T_1 and T_2 of the minimum maintenance cost per unit time are 380 and 760 respectively, and the corresponding value is 85.7015 yuan per day. Select the best part of the value near T_1 and T_2 to compare, as shown in Table 2.

Table 2. Maintenance cost per unit time for different preventive maintenance times.

		T_1						
		365	370	375	380	385	390	395
T_2	750	86.0539	85.8757	85.7960	85.8147	85.9316	86.1465	86.4589
	755	86.0939	85.8683	85.7403	85.7100	85.7770	85.9413	86.2025
	760	86.2281	85.9557	85.7802	85.7015	85.7195	85.8339	86.0446
	765	86.4573	86.1390	85.9167	85.7904	85.7600	85.8254	85.9862
	770	86.7827	86.4191	86.1508	85.9777	85.8997	85.9166	86.0283

5. Conclusion

By introducing time constraints into product warranty period, a model of product maintenance cost per unit time considering time constraints is established for the first time in this paper. The optimal time for incomplete preventive maintenance within the constrained time is obtained through

case study and cost considerations, which verifies the availability and effectiveness of the model. This model can be used for reference to optimize the current large-scale equipment maintenance service cost, enhance the rationality of the maintenance cost formulation during the warranty period, and have a strong reference value for manufacturers and users.

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